TII Technology Innovation Institute

Magic and other shadow arts

Ingo Roth, Nov 18th 2024

joint work with


Raphael Brieger


Markus Heinrich


Martin Kliesch

Stability of classical shadows under gate-dependent noise

arXiv:2310.19947

Quantum device characterization

Extract information about a quantum device from observation

VALIDATION

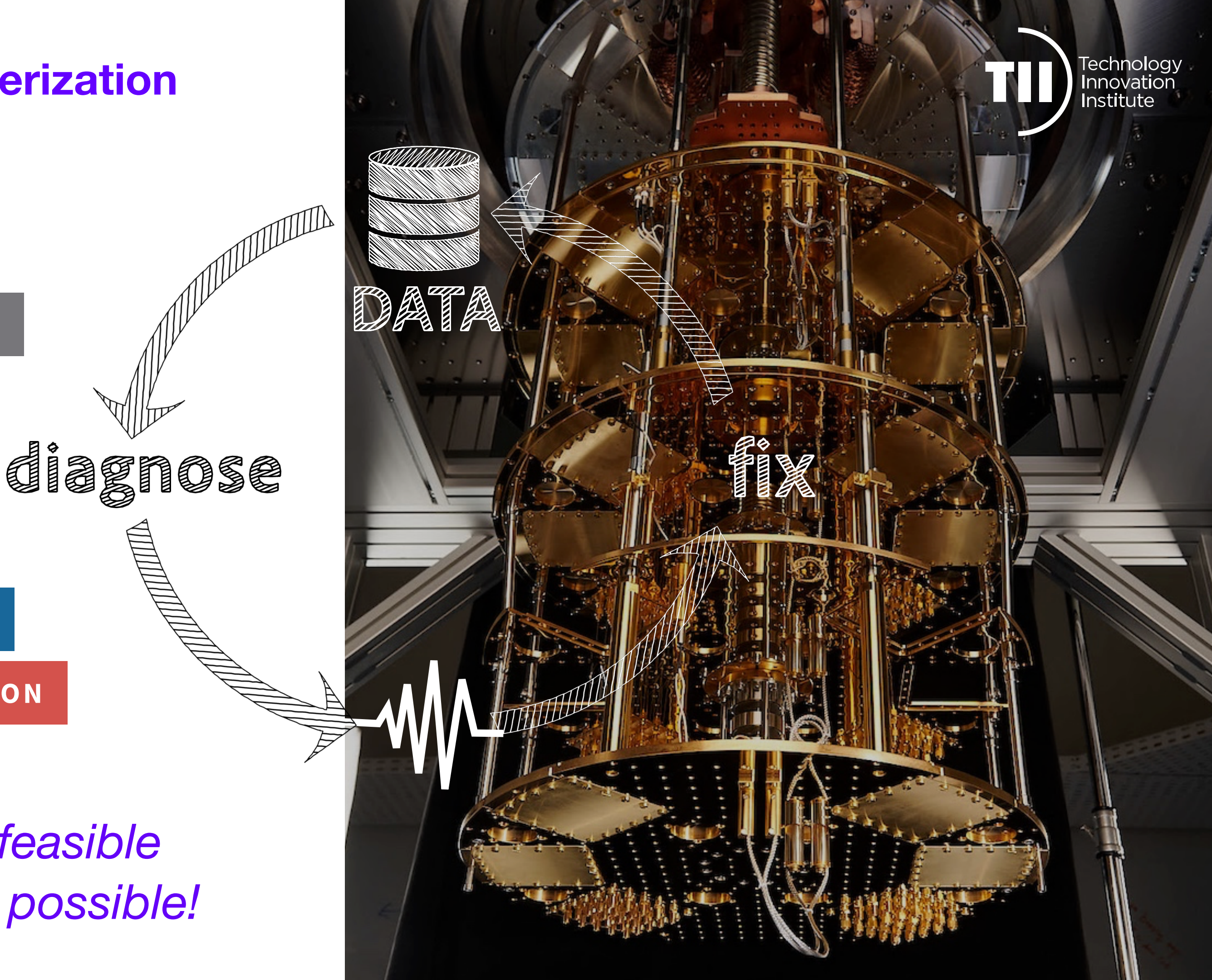
BENCHMARKING

CERTIFICATION

ESTIMATION

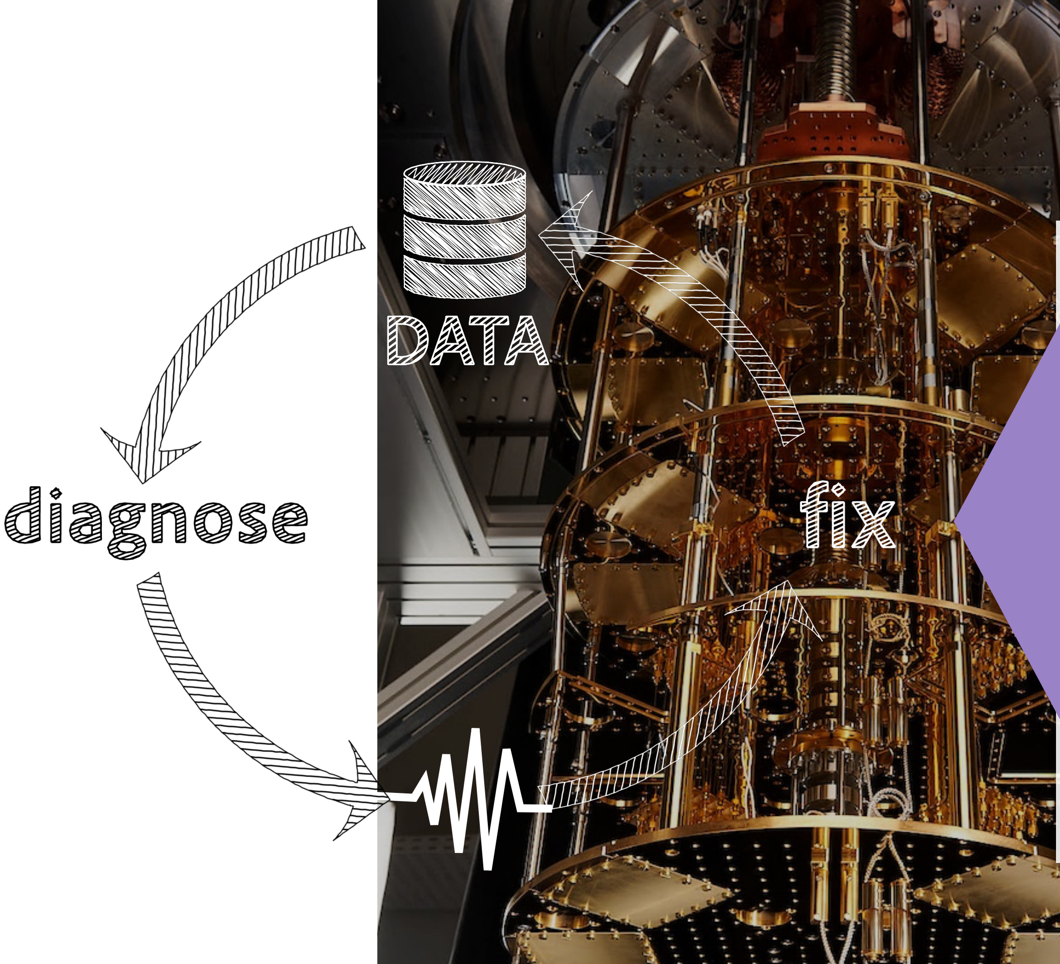
LEARNING

IDENTIFICATION



Learn as much as *feasible*
… ***use*** as much as *possible!*

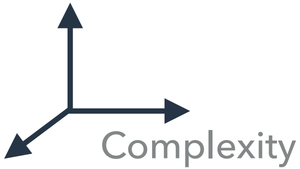
… ***use*** as much as *possible!*



- Building trust
- Benchmark fabrication & inform design
- Understand noise and errors
- Calibrate control & tune-up
- Noise mitigation
- Noise-aware compilation / algorithms

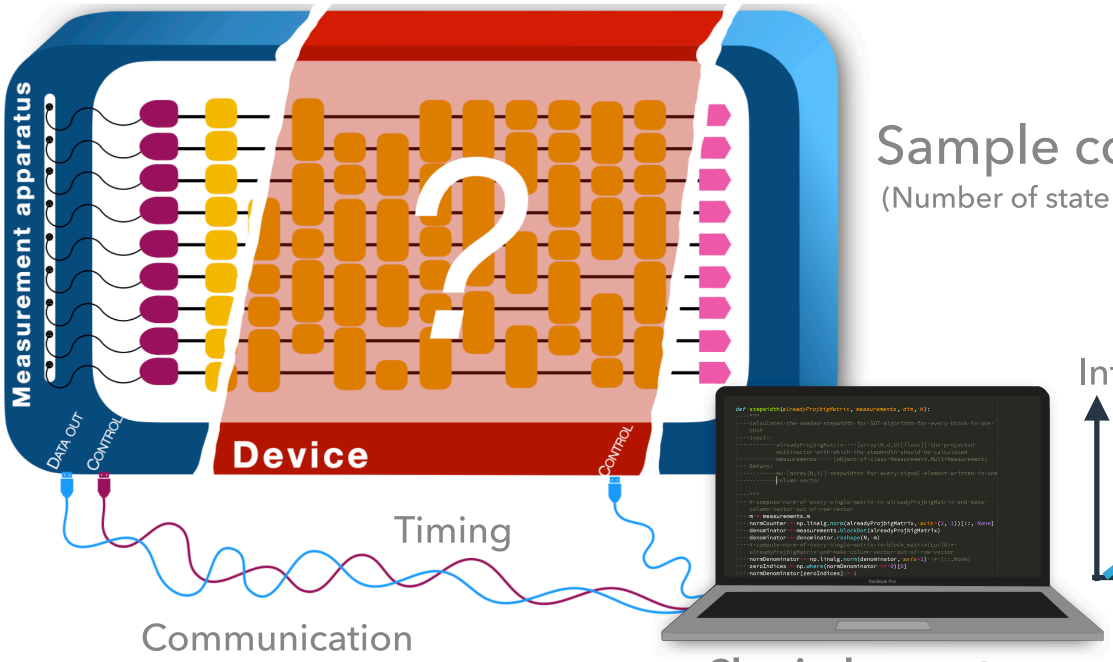
Theory — QIBO — **Experiment**

Eisert, Hangleiter, Walk, **IR**, Markham, Parekh, Chabaud, Kashefi. *Nat. Rev. Phys.* 2 (2020), Kliesch, **IR**. PRX *Quantum* 2 (2021)



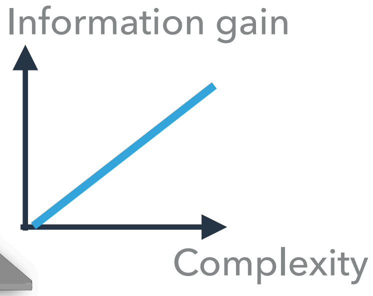
Number of measurement settings

Measurement complexity



Sample complexity

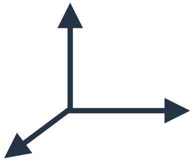
(Number of state copies / channel invocation)



Classical computer

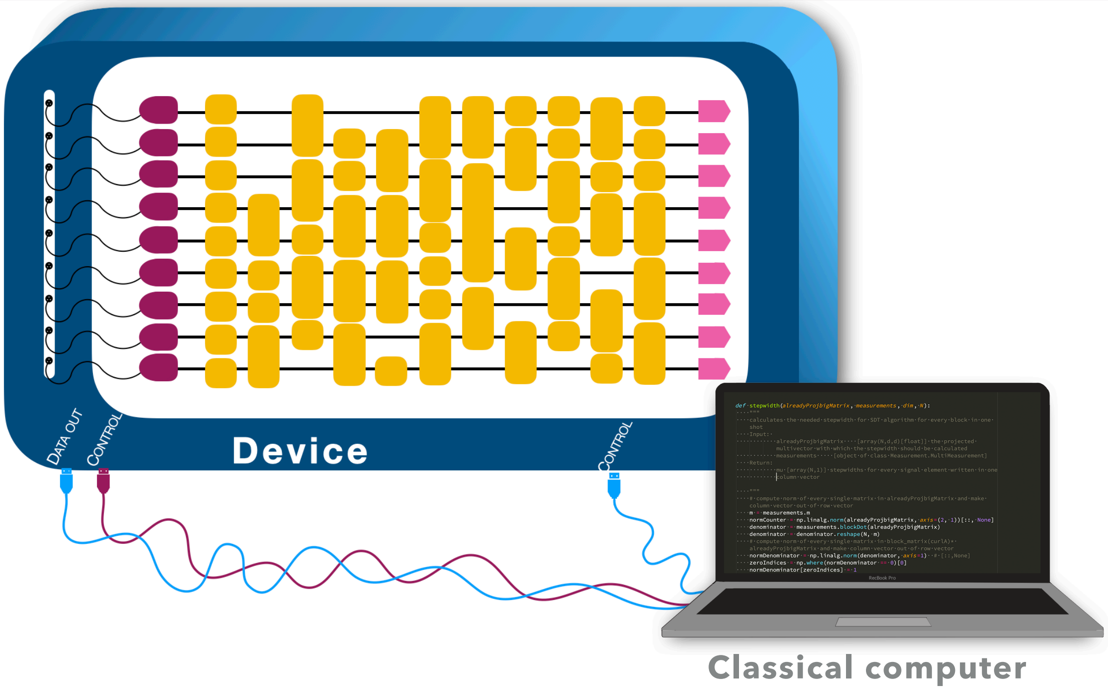
Memory **Runtime**

MEASURES OF COMPLEXITY



Assumption

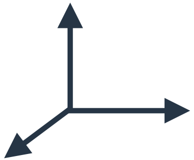
ANATOMY OF CHARACTERIZATION PROTOCOLS



device-independent

device-dependent

ANATOMY OF CHARACTERIZATION PROTOCOLS



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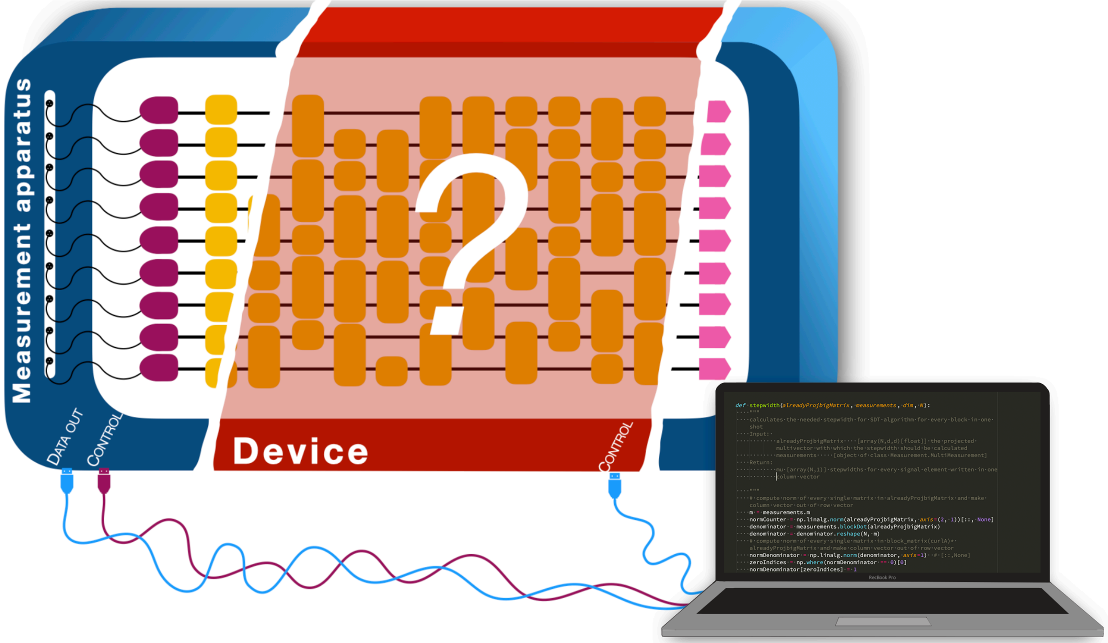

Classical computer

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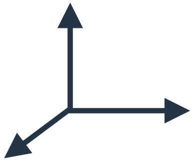

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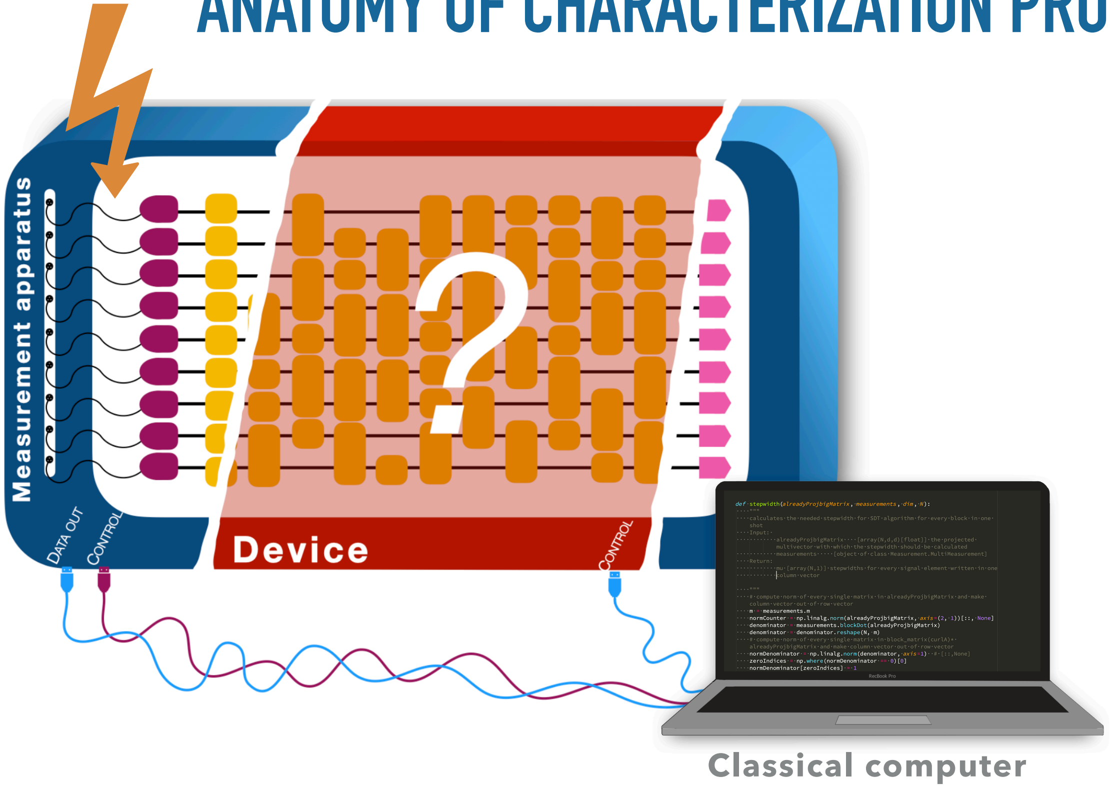
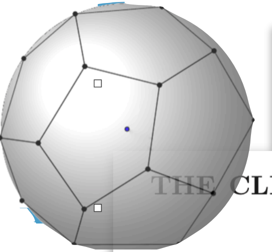


Quantum characterization and randomness and non-stabilizerness



Exact and approximate unitary 2-designs and their application to fidelity estimation

Christoph Dankert, Richard Cleve, Joseph Emerson, and Etera Livine
Phys. Rev. A **80**, 012304 – Published 6 July 2009

2-design

THE CLIFFORD GROUP FORMS A UNITARY 3-DESIGN

ZAK WEBB[1] Quantum Information & Computation, 2016

3-design

PHYSICAL REVIEW A **96**, 062336 (2017)

Multiqubit Clifford groups are unitary 3-designs

Huangjun Zhu*

Low rank matrix recovery from Clifford orbits

Richard Kueng, Huangjun Zhu, David Gross

Clifford 4-design

The Clifford group fails gracefully to be a unitary 4-design

Huangjun Zhu[1], Richard Kueng[1], Markus Grassl[2], and David Gross[1]

Clifford 4-design

Tight informationally complete quantum measurements

A J Scott[1]

Published 11 October 2006 • 2006 IOP Publishing Ltd

Journal of Physics A: Mathematical and General, Volume 39, Number 43

2-design

Informational completeness & tightness

Optimizing quantum process tomography with unitary 2-designs

A J Scott

Published 23 January 2008 • 2008 IOP Publishing Ltd

Journal of Physics A: Mathematical and Theoretical, Volume 41, Number 5

2-design

Robust Extraction of Tomographic Information via Randomized Benchmarking

Shelby Kimmel, Marcus P. da Silva, Colm A. Ryan, Blake R. Johnson, and Thomas Ohki
Phys. Rev. X **4**, 011050 – Published 25 March 2014; Erratum Phys. Rev. X **6**, 029902 (2016)

2-design

Recovering Quantum Gates from Few Average Gate Fidelities

I. Roth, R. Kueng, S. Kimmel, Y.-K. Liu, D. Gross, J. Eisert, and M. Kliesch
Phys. Rev. Lett. **121**, 170502 – Published 24 October 2018

Clifford 4-design

Optimal measurement setting

Low rank matrix recovery from rank one measurements

4-design

Richard Kueng [a], Holger Rauhut [b], Ulrich Terstiege [b]

Multiqubit randomized benchmarking using few samples

Jonas Helsen, Joel J. Wallman, Steven T. Flammia, and Stephanie Wehner
Phys. Rev. A **100**, 032304 – Published 3 September 2019

Clifford 4-design

Fast state tomography with optimal error bounds

M Guţă[4,1], J Kahn[2], R Kueng[3] and J A Tropp[3]

Published 28 April 2020 • © 2020 IOP Publishing Ltd

3-design

Predicting many properties of a quantum system from very few measurements

Hsin-Yuan Huang, Richard Kueng & John Preskill

3-design

Single-shot estimators

Shadow estimation of gate-set properties from random sequences

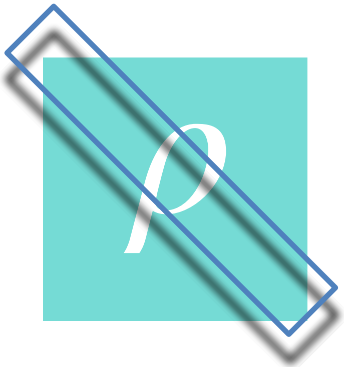
Nature Communications **14**, Article number: 5039 (2023)

J. Helsen, M. Ioannou, J. Kitzinger, E. Onorati, A. H. Werner, J. Eisert & I. Roth

3-design

Classical shadows & IC POVMs

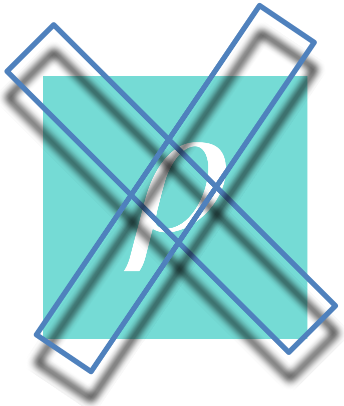
$$\{\Pi_x = |x\rangle\langle x|, \quad x \in \mathbb{F}_2^n\}$$



Classical shadows & IC POVMs

$$\{\Pi_x = |x\rangle\langle x|, \quad x \in \mathbb{F}_2^n\}$$

$$\{\Pi_{x,g} \propto g\,|x\rangle\langle x|\,g^\dagger, \quad x \in \mathbb{F}_2^n, g \in G\}$$



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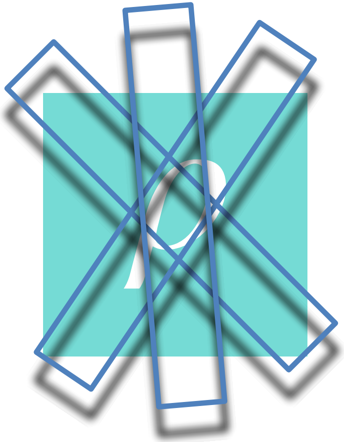

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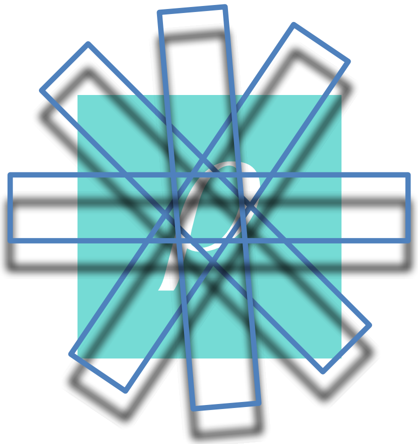

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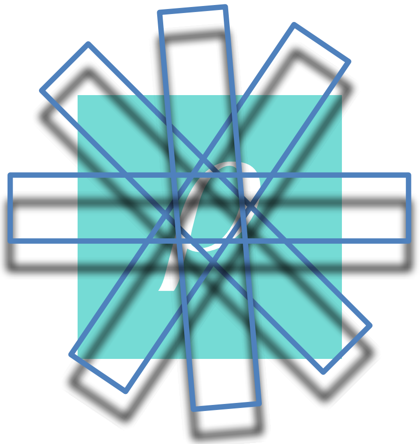


[Scott 2006]

A *positive-operator valued measure (POVM)* is **informationally complete (IC)** if

$$S = \sum_{g,x} |\Pi_{x,g})(\Pi_{x,g}|$$

is **invertible.**

Tight frames and 2-designs

$$S = \left[\begin{array}{c|ccc} 1 & & & \\ \hline & \frac{1}{d+1} & & \\ & & \ddots & \\ & & & \frac{1}{d+1} \end{array}\right]$$

G a **unitary 2-design**

Here … G sub-set of the **Clifford group**!

Classical shadows … surprisingly efficient

Protocol: [Scott 2006], …

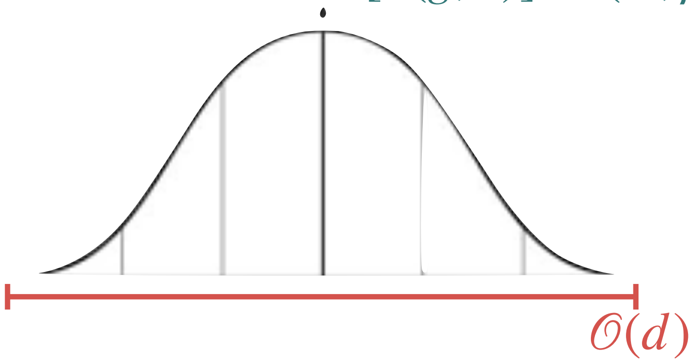
1) Measure $\{\Pi_{x,g}\}$ on state ρ

2) Calculate and average

$$\hat{o}(g,x) = (O\,|\,S^{-1}\,|\,\Pi_{x,g}) \in \mathcal{O}(d)$$

By construction …

$\mathbb{E}[\hat{o}(g,x)] = (O,\rho)$



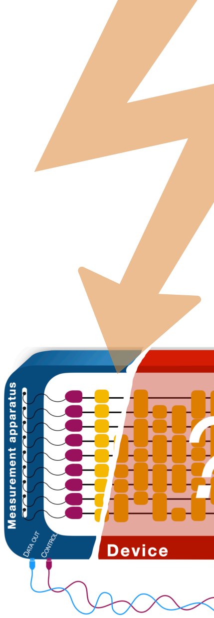
Theorem: For $O = |\psi\rangle\langle\psi|$ and G a 3-design,

$$\mathrm{Var}[\hat{o}] \in \mathcal{O}(1).$$

[Huang, Kueng & Preskill 2020]

✓ Global Clifford rotations

Classical shadows ... what could possibly go wrong?



Noisy-frame operator:

Gate-dependent noise!

$$\tilde{S} = \sum_{x,g} |\Pi_{x,g})(\Pi_{x,g}|\Lambda(g)$$

Biased estimator:

$$\mathbb{E}[\hat{o}] = (O, \rho) + (d+1)(O_0|S - \tilde{S}|\rho)$$

Large bias?!?

Definition :

$$\text{bias} \sim \kappa(d) \times \text{implementation error}$$

$\kappa(d) \in \mathcal{O}(1)$	otherwise	$\kappa(d) \in \Omega(d^{1/c})$
"stable"	"instable"	"Really instable"

e.g. naive bound :

$$\text{bias} \leq (d+1) \max_{g \in G} \|\text{id} - \Lambda(g)\|_\diamond$$

Instability . . .

Let $O = |H\rangle\langle H|^{\otimes n}$ with $|H\rangle \propto |0\rangle + \mathrm{e}^{\mathrm{i}\pi/4}|1\rangle$

and G **local Clifford** unitaries.

Fidelity estimation with local shadows.

There exist $\Lambda_\epsilon(g) = (1-\epsilon)\,\mathrm{id} + \epsilon\Lambda(g)$

gate-dependent noise such that

such that

$$\mathrm{bias} = \kappa\epsilon \quad \text{with} \quad \kappa \in \Omega(d^{1/4})$$

“Really instable”

Are there classes of

stable shadow estimation

settings?

Maybe less magic, please.

Stability . . .

Definition :

$$\|O\|_{\mathrm{st}} := \frac{1}{d} \sum_{a \in \mathbb{F}_2^{2n}} |(\sigma_a | O)|$$

"Stabiliser Renyi 1/2-entropy"

Lemma: $G \subset \text{Clifford}$, it holds

Ideal **Pauli-diagonal** channel

$$\tilde{S} = \frac{1}{d} \sum_{a \in \mathbb{F}_2^{2n}} s_a | \sigma_a)(\sigma_a | \bar{\Lambda}_{a'}$$

with $s_a \in (0,1]$ and $\bar{\Lambda}_a$ quantum channels.

Theorem: $G \subset \text{Clifford}$, it holds

$$\text{bias} \leq \boxed{\|O\|_{\mathrm{st}}} \max_{g \in G} \| \mathrm{id} - \Lambda(g) \|_\diamond$$

Scaling function $\kappa(d)$ **Implementation error** ϵ

This bound is tight. Similar control over the variance.

Stable settings, e.g.

- ☑ Pauli observables
- ☑ Stabiliser states
- ☑ Bounded-degree local Hamiltonians

And 'robust' classical shadows? they are robust, right?

$$\tilde{S} = \sum_{x,g} |\Pi_{x,g})(\Pi_{x,g}|\Lambda(g) = \sum_{x,g} |\Pi_{x,g})(g\Lambda^\dagger(\Pi_x)g^\dagger|$$

Gate-independent left noise!

$\implies$

Symmetry is preserved under ***noise!***

$$S = \begin{bmatrix} 1 & & & \\ & \tilde{p} & & \\ & & \ddots & \\ & & & \tilde{p} \end{bmatrix}$$

Idea: **Empirically estimate** parameter $\tilde{p}$ using fidelity estimation for **trusted input state.** [Chen et al. 2021]

'Robust' classical shadow protocol

"Really instable"

There exist **gate-dependent noise** such that

$$\Lambda_\epsilon(g) = (1-\epsilon)\ \mathrm{id} + \epsilon\Lambda(g)$$

$$\mathrm{bias}_{\text{robust shadow}} \geq \langle O_0 \rangle \Omega(de^\epsilon)$$

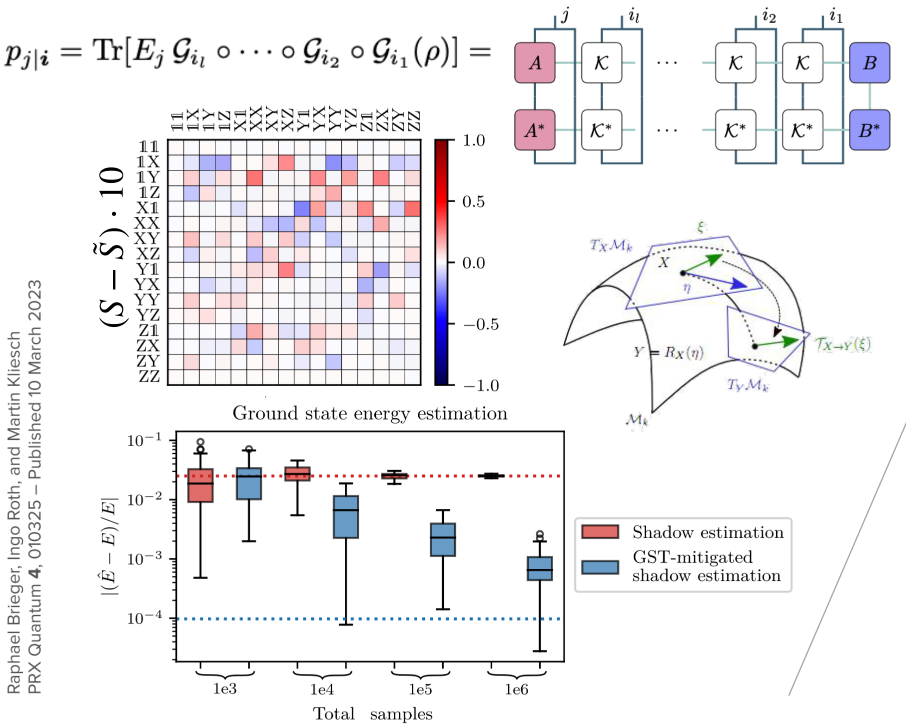
However!

Requires **Pauli-spikiness** of effective noise and **support of observable** to **conspire**!

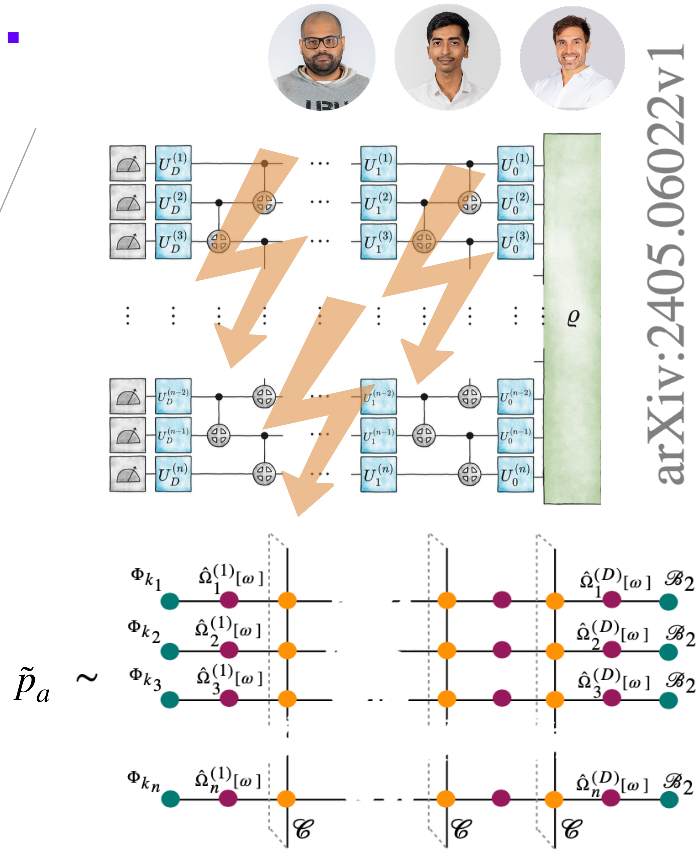
Stable, e.g. for *Pauli-**isotropic** noise*

More robust classical shadows . . .

Compressive Gate-set tomography for robust shadows

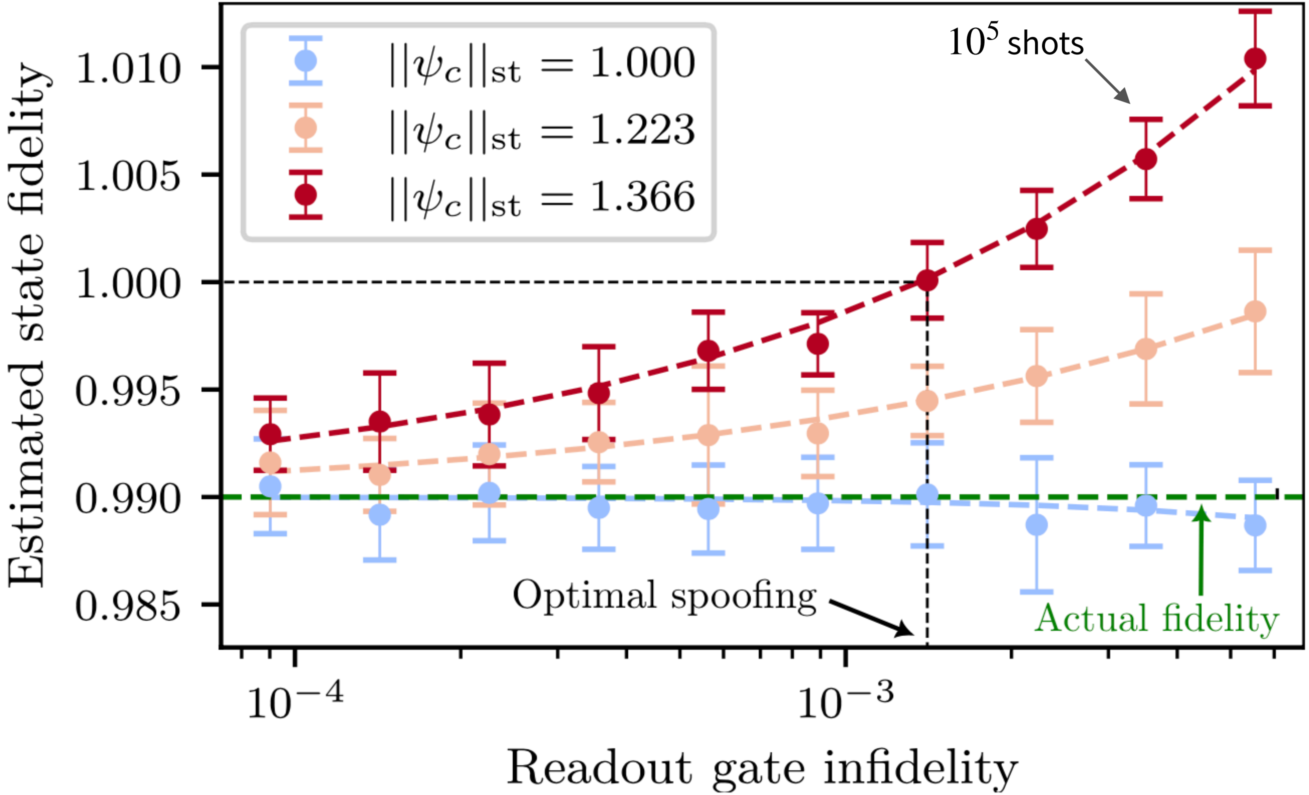


Compressive Gate Set Tomography
Raphael Brieger, Ingo Roth, and Martin Kliesch
PRX Quantum **4**, 010325 – Published 10 March 2023



Robust super-shallow circuits

Consistent under-rotation . . .



$\pi/2$**-pulse implementation** of **fidelity estimation** on *single qubit*

Summary

- **Semi-device dependence** / **robustness** is important

- *In case you were worried*, classical **shadows** are surprisingly **robust** for ‘**non-magic’ observables**

- **Mitigation can** sometimes even **do harm** for *gate-dependent noise*

Stability of classical shadows under gate-dependent noise

- Noise **assumption** *might be to simplistic!*
  More studies with **gate-dependent noise & beyond!**

arXiv:2310.19947